Name ____ Test 2, Spring 2021

1) Given the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$ and $\vec{x}_S = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_S$, find a formula for $[\vec{x}]_B$. (10 points)

2) Given the bases $B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 13 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$, find a formula for the change of basis matrix that converts vectors from basis B_1 into vectors from basis B_2. (10 points)

3) Find the determinant of the matrix below. (15 points)

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -4 \\ 3 & 4 & 0 & 2 \end{bmatrix}$$

4) Given the linear transformation $T\colon \mathbb{R}_S^2 \to \mathbb{R}_S^2$ given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_S\right) = \begin{bmatrix} 3x_2 \\ x_1 + x_2 \end{bmatrix}_S$ and the bases below, find a formula for $\left[T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{B_1}\right)\right]_{B_2}$. (10 points)

$$B_1 = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}\right\}$$
$$B_2 = \left\{\begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right\}$$

5) Answer the following questions. (3 points each)

A) Let A be a 3×3 matrix and assume that it has rank 2. How many solutions does $A\vec{x} = \vec{0}$ have?

B) Let A be a 3×4 matrix and assume that the corresponding linear transformation T is not onto. What is the minimum dimension of the null space of A?

C) Let A be a 3×7 matrix. Assume that the dimension of the row space is 3. What is the dimension of the column space?

D) Consider a system of 5 equations in 3 variables. Assume there are infinitely many solutions. If A is the matrix representing this system, what are the possible values for the rank of A?

E) Let A be a 6×6 matrix and T the corresponding linear transformation. If $\dim(\ker(T)) = 2$, what is the rank of A?

6) Find the product below. (10 points)

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$$

7) Find the quadratic form that comes from the matrix below. (5 points)

$$\begin{bmatrix} 1 & 4 \\ 4 & 7 \end{bmatrix}$$

8) True or false? The matrices below are inverses of each other. (5 points)

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

9) Given the information below, find a formula for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. (5 points)

$$\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix}$$

$$\begin{aligned} 2x_1 + 2x_2 &= 5 \\ x_1 - 6x_2 &= 7 \end{aligned}$$

10) Find the row space of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

11) What is the rank of the matrix below? (5 points)

$$\begin{bmatrix} 1 & 0 & 4 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

12) Given the information below find a formula for $[I]_{B_1}^{B_2}$, the change of basis matrix that converts vectors in V_{B_1} to corresponding vectors in basis W_{B_2}. (5 points)

$$V_{B_1} = \text{span}(B_1)$$
$$W_2 = \text{span}(B_2)$$

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} \right\}$$

$$B_2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \right\}$$